Project: IEEE P802.15 Working Group for Wireless Personal Area Networks (WPANs)

Submission Title: [Theoretical Size Constraints for Antennas based on Quality Factor Q"]
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Abstract: [This presentation describes the classical theoretical limitations for antenna size based on seminal work by several researchers. Antenna volume or "size" can be related to antenna "Q" (Quality Factor) based on the so-called Chu-Harrington Limit. The "Q" is generally accepted to be a measure of antenna bandwidth, although describing a precise relationship for UWB is difficult. Nevertheless, an understanding of the general trend described by such theoretical considerations can contribute to the successful design of portable ultra-wideband devices)]

Purpose: [General Release]

Theoretical Size Constraints for Antennas based on Quality Factor "Q"

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UWB Outlook

- The recent FCC approval of spectrum for ultra-wideband (UWB) commercial usage will lead to the development of a new class of wireless devices and applications.
- Development of these devices will place demands on antenna technology, with specific emphasis on broadband operation and miniaturization

UWB Future Concerns

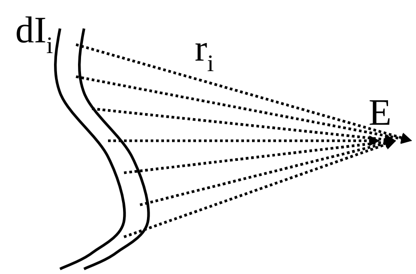
- Size, Form Factor, and Cost will become dominant design considerations.
- The ultimate limitations of the antenna size, bandwidth, and gain are determined from basic theoretical considerations and can influence the design of any portable system.

Antenna Engineers Dilemma

- “Bandwidth and Antenna Size are inversely related”
  - The bandwidth of a “small antenna” can be increased by decreasing the efficiency, thereby reducing the gain.

- “Antenna Gain and Size are directly related”
  - The maximal gain of an antenna can be enhanced somewhat by varying the geometry and the bandwidth can be similarly increased
  - Radiated E-Field from a short Antenna:

$$\left|\vec{E}\right| \propto \sum_i \frac{1}{r_i} \frac{d\,\vec{I}_i(t - r_i/c)}{dt}$$



  - For a fixed current, greater wire length L increases the field strength. For greater E, increase I or L.

Historical Research:Antenna Size Limitations

- Size versus bandwidth capability of an antenna first introduced by Harold Wheeler and L. J. Chu, and later by Roger Harrington.
- In 1946, Wheeler[1] introduced the concept of the “radiansphere” and the volume relation to the maximum power factor achievable by the antenna.
- The power factor, it is reasoned, is proportional to the antenna volume and also a shape factor. The nominal bandwidth is given as the power factor *p* multiplied by the resonance frequency f_0.

$$V \propto p \propto \frac{\Delta f}{f_0}$$

(p=antenna resistance/antenna reactance)

[1]Wheeler, Harrold, " Small Antennas," IEEE A&P, , AP-23, 4, pp.462-469, 1975

Theoretical Development History

- In 1948, Chu[2] extended Wheeler's analysis and expressed the fields for an omnidirectional antenna in terms of spherical wave functions and found limits for the minimum antenna quality factor (Q), the maximum gain (G_m) and the ratio G/Q.
- used a partial fraction expansion of the wave impedance of spherical modes that exist outside the smallest circumscribing sphere surrounding the antenna to obtain an equivalent ladder network from which the Q could be found by conventional circuit analysis.

[2]"Physical Limitations of Omni-Directional Antennas," L.J. Chu, J. Appl. Phys., v.19, pp1163-1175, 1948.

Theoretical Development History

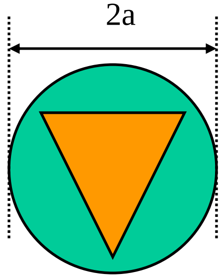
- in 1959, Harrington[3] related the effects of antenna size, minimum Q, and gain for the near and far field diffraction zones for linearly and circularly polarized waves, and also treated the case where the antenna efficiency η is less than 100%.
- In 1964, Collin and Rothschild[4] presented a method to find the minimum Q without using the equivalent network for both spherical and cylindrical modes.
- In 1969, Fante[5] extended these results to multimode antennas.

[3]Harrington, R. “Effects of Antenna Size on Gain, Bandwidth, and Efficiency," J, Nat. Bur. Stand. , v 64-D, pp. 1-12, 1960
[4]Collin, R.E. and S. Rothchild, “Evaluation od Antenna Q,” IEEE Trans. AP, AP-12, pp.23-27, 1964
[5]Fante, R.L., “Qulaity Factor of general Ideal Antennas,” IEEE Trans AP, AP-17, pp. 151-157, 1969

Theoretical Development History

Additional work[6] carried out from 1969 through 1996 obtained exact expressions for antenna Q over an expanded size range. These efforts led to the equation for the minimum Q:

$$Q = \frac{1}{(ka)^3} + \frac{1}{ka}$$



"a" is the radius of a sphere enclosing the antenna. "k" = $2\pi/\lambda$. A lowest order single mode (TM or TE) is implied

[6]McClean, " A Re-examination of the Fundamental Limits on the Radiation Q of Electrically Small Antennas," IEEE Trans AP, v44, pp. 672-675, May 1996.

What is Q for an Antenna

- Q is a parameter that describes energy transfer in oscillating systems
  - Q= 2π x (standing peak energy per cycle/average power lost per cycle)
  - For antennas (resonant and non-resonant) the accepted definition is:

    Q= $2\omega W_{avg}/P_{rad}$ for antennas, where the average per cycle is taken of the stored (evanescent-wave) magnetic and electric field energies (W), and P is the radiated power per cycle.

Why is Q of Interest

- For the narrowband case antenna Q can be simple related to the fractional bandwidth BW:

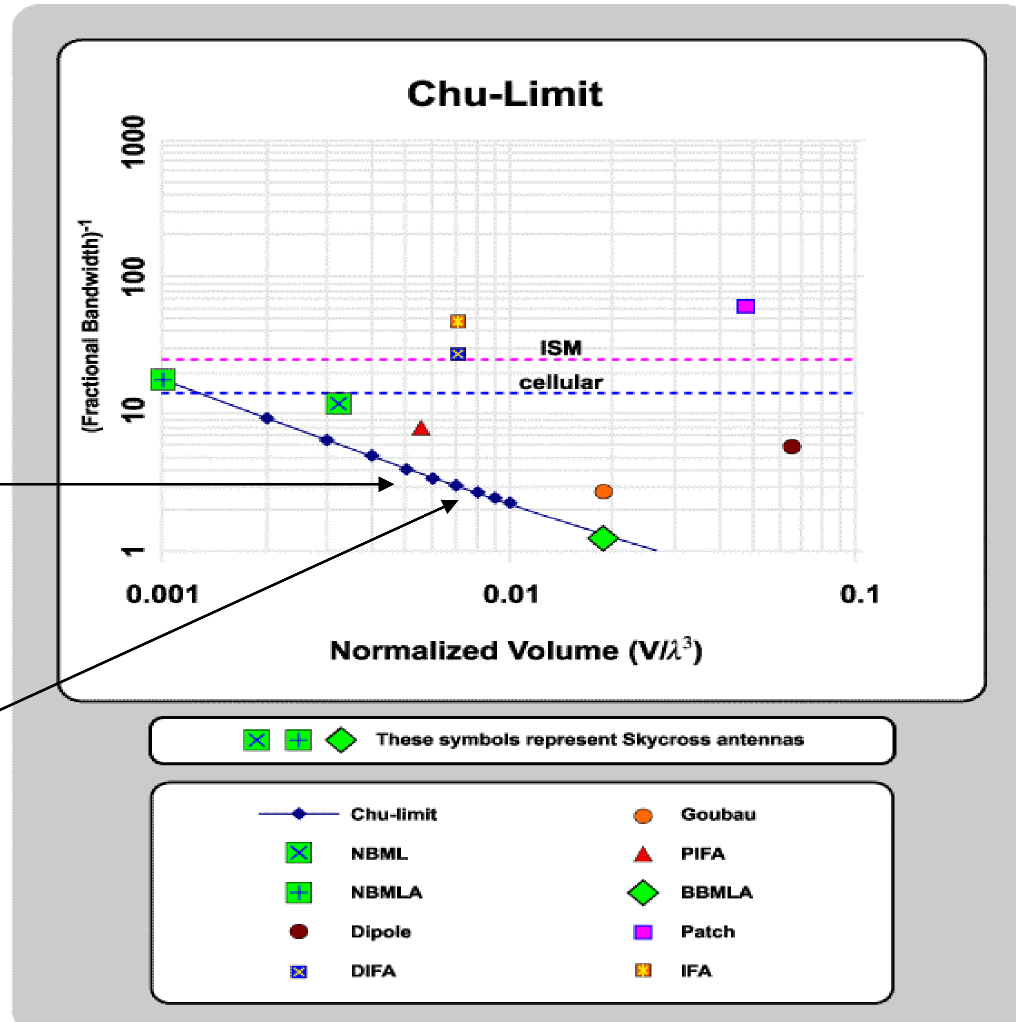
$$\frac{1}{BW} = \frac{f}{\Delta f} \cong Q = \frac{1}{ka} + \frac{1}{(ka)^3}$$

- For the wideband case, the relationship is not easily described, but the trend still exists

Not all Antennas are Minimum Size

- Antenna volume V can be ~related to BW
- V is normalized to λ^3
- Antennas having 100% efficiency remain above the limit curve
- Some antenna designs are smaller than others

$$(BW)^{-1} = \frac{1}{6\pi^2 V} + \frac{1}{2\pi\left(\frac{3}{4\pi}\right)^{1/3} V^{1/3}}$$

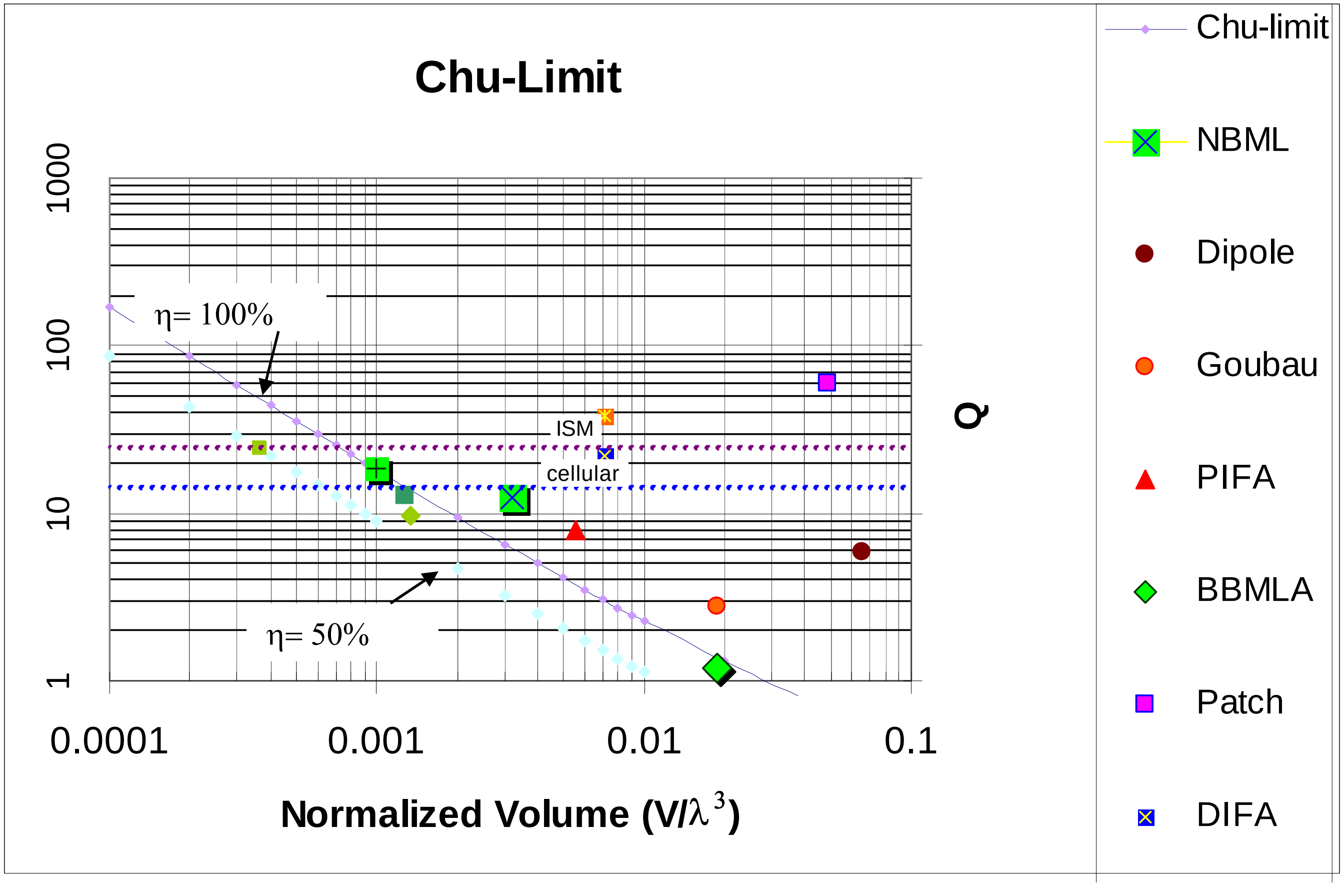


Efficiency Regulates the Size Limit

- **Lesser efficiency** (η<100%) ➡
  - **provides greater bandwidth for a given size**

  OR

  - **provides for a smaller antenna for a given bandwidth**

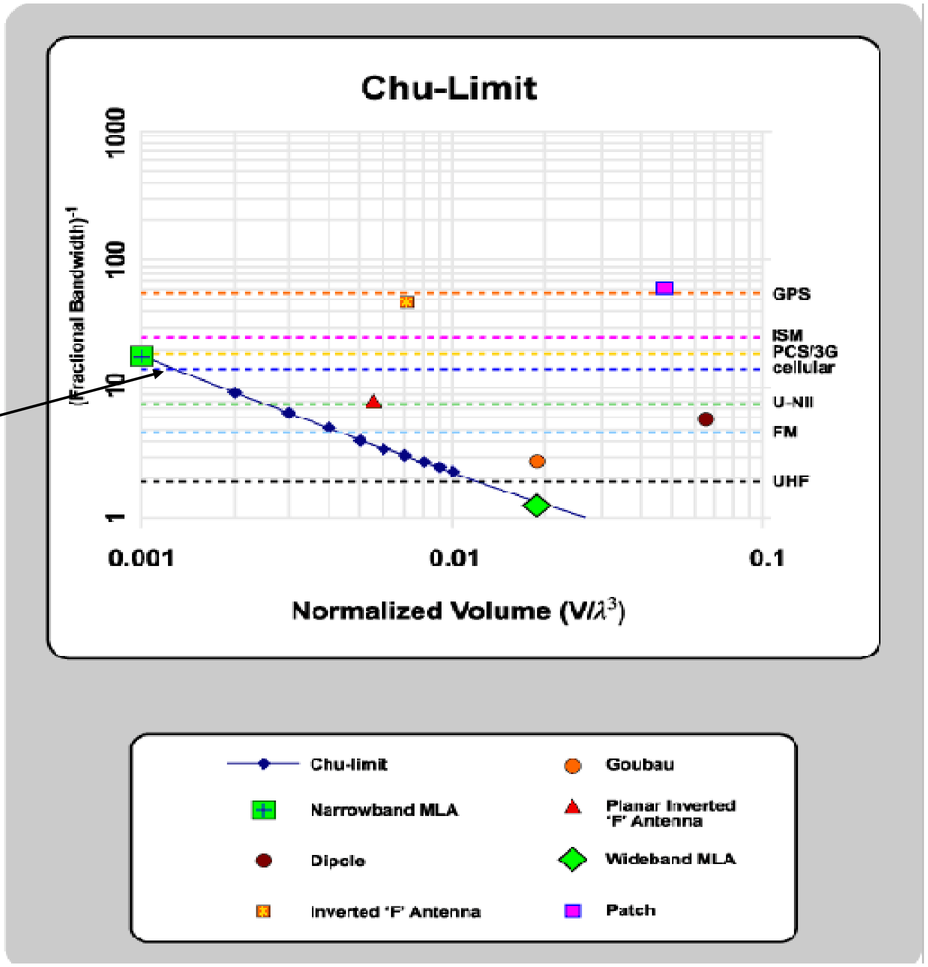


Goal is to have antenna design that is on the limit curve for a fixed η

Example Calculation

Sample Narrowband Calculation

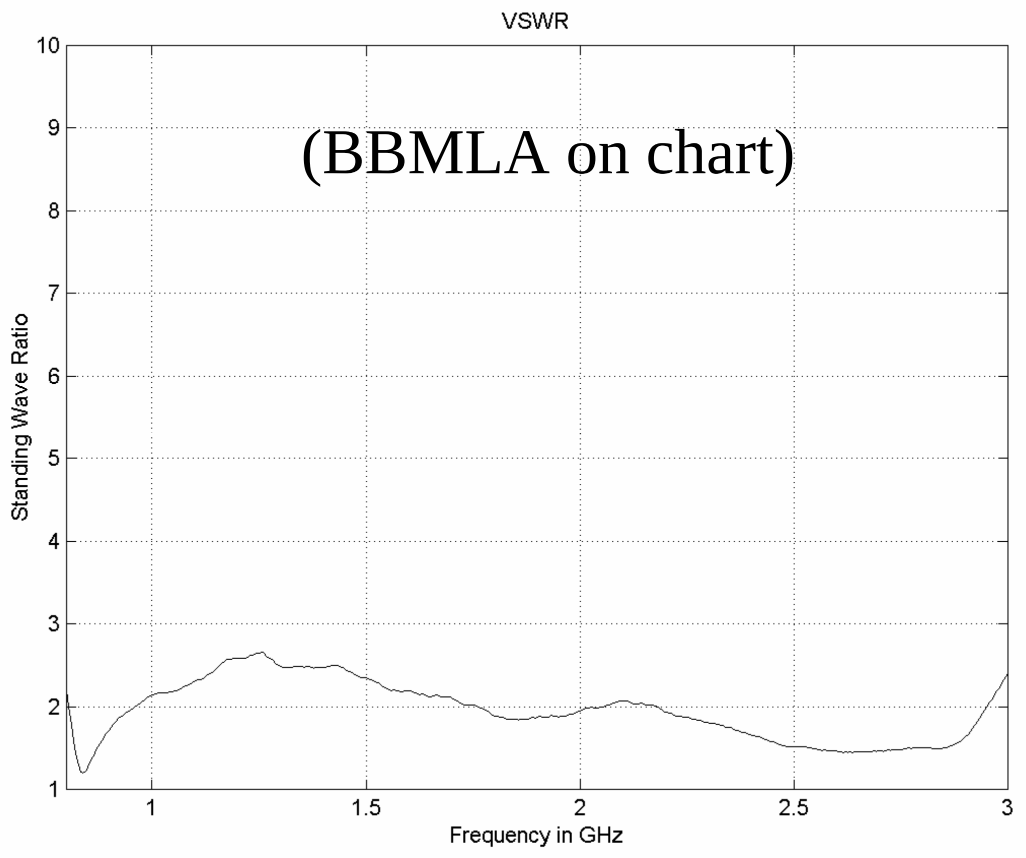
- PCS Bandwidth:1850-1990 MHz: (1990-1850)=140 MHz Q~(1990+1850)/2(140)= 13.7
- Smallest allowed volume for 100% efficiency (from graph) = 0.0015
- V_{min}= 0.0015λ^3 (λ = 3x10^{11}/1920 x10^6=156 mm)
- V_{min}= 5600 mm^3 or an equivalent sphere of 11 mm radius
- Model 1 Antenna volume:(η=77%) 25 x 20 x 8 mm = 4000mm^3



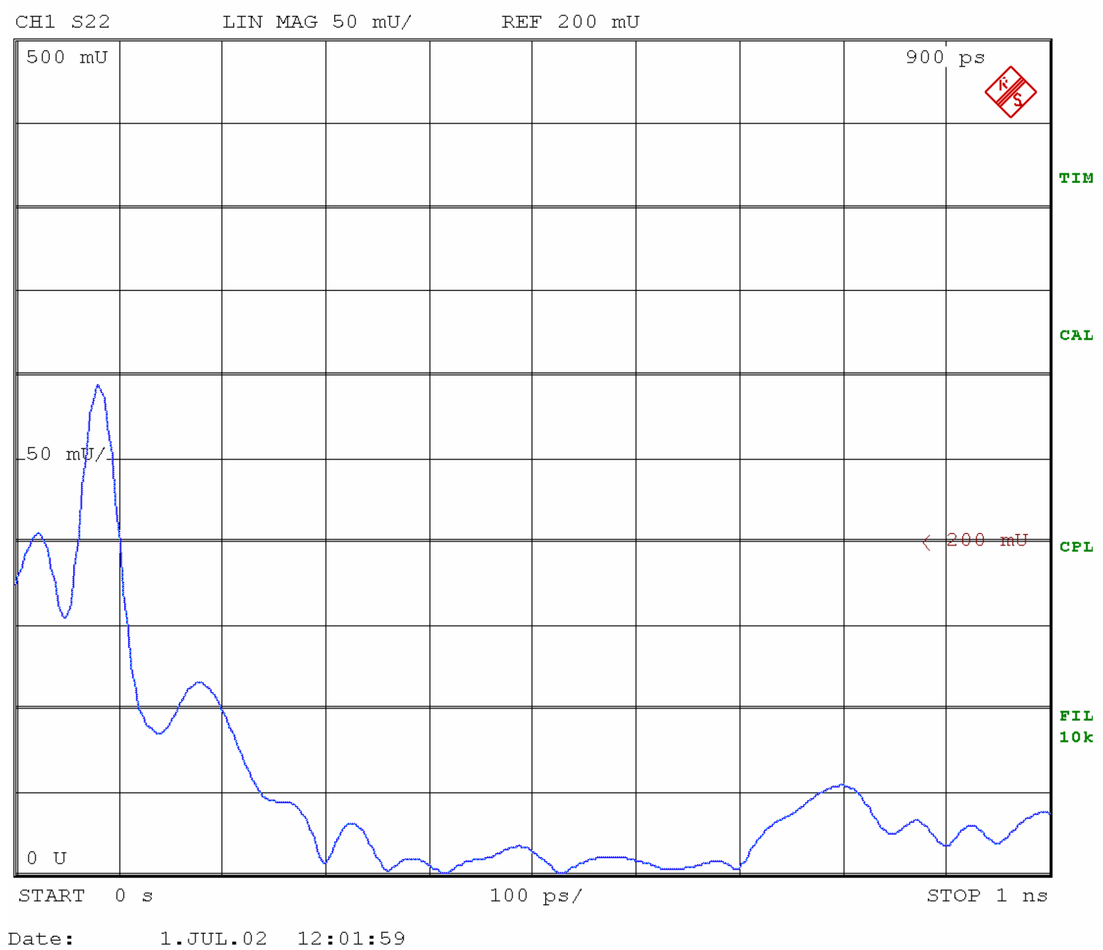
Q for Broadband Antennas

- Relating Q to operating bandwidth for extremely wideband systems is inexact
- Experience shows that $Q \sim [f_L x\ f_h]^{1/2}/\Delta f$ for Q>~2. Q~1.414 for an octave bandwidth.

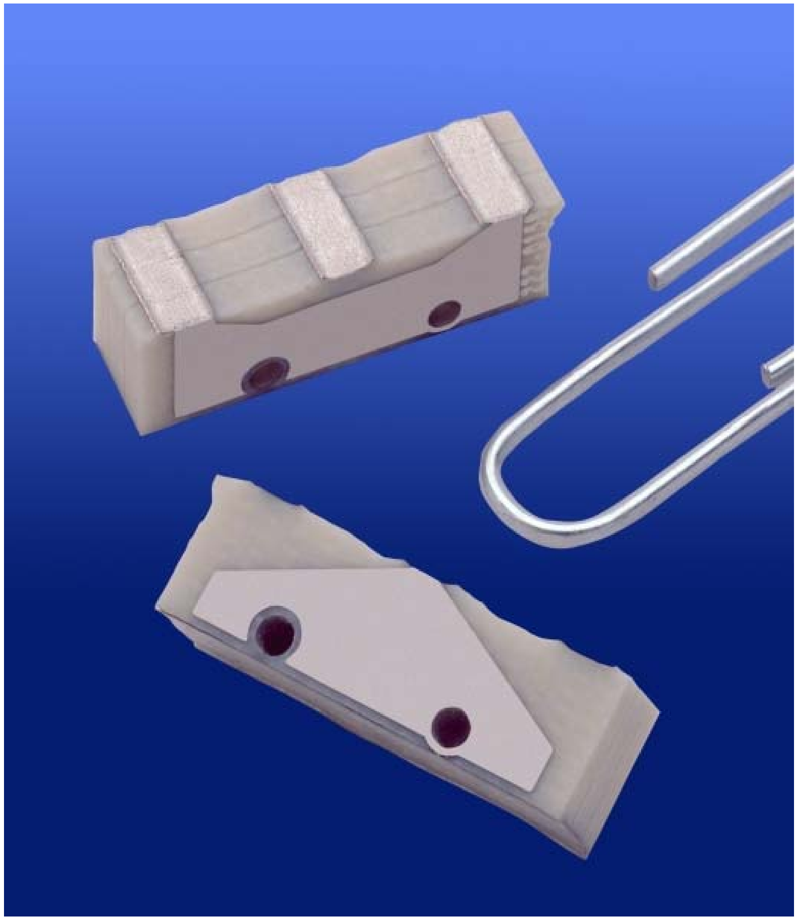
Example UWB Antenna



Example UWB Impulse Response



Example surface mount broadband antenna (5-6 GHz) using printed circuit board construction techniques to achieve 3-dimensional MLA structure



Summary

- Antenna size and bandwidth follow general trends established by well-established theory
- The small sizes required for portable devices can take advantage of antennas operating at higher “center” frequencies through reductions in fractional bandwidth (BW)--a system design issue
- Not all antennas offer the smallest size-bandwidth tradeoff or efficiency--choose a well designed antenna for the application!